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Code No. : 14122

VASAVI COLLEGE OF ENGINEERING (Autonomous), HYDERABAD

B.E. (CBCS) IV-Semester Main Examinations, January-2021

Numerical Methods, Probability and Statistics

(Common to Civil, EEE & Mech.)

Time: 2 hours

Max. Marks: 60

Note: Answer any NINE questions from Part-A and any THREE from Part-B

Part-A (9 × 2 = 18 Marks)

Q. No.	Stem of the question	M	L	CO	PO										
1.	Evaluate $\Delta^2(e^x)$, taking $h = 1$.	2	2	1	1,12										
2.	Find the second divided difference of $f(x) = \frac{1}{x}$ using the points a, b, c .	2	2	1	1,12										
3.	Write the expressions for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ using Newton's backward interpolation formula.	2	1	2	1,12										
4.	Using Euler's method, find the approximate value of $y(0.1)$ for the initial value problem $y' = x^2 + y^2, y(0) = 1$.	2	3	2	1,12										
5.	Define discrete and continuous random variables with an example.	2	1	3	1,12										
6.	If the probability density function of a continuous random variable X is $f(x) = ae^{- x }, -\infty < x < \infty$, then find the value of a .	2	2	3	1,12										
7.	Define (i) null hypothesis and (ii) alternate hypothesis.	2	1	4	1,12										
8.	Write short note on type I and type II errors.	2	1	4	1,12										
9.	Test whether the equations $x = 4y + 5$ and $y = \frac{1}{8}x + 4$ represent valid regression lines.	2	2	5	1,12										
10.	Show that the coefficient of correlation is the geometric mean of regression coefficients.	2	2	5	1,12										
11.	State Lagrange's interpolation formula.	2	1	1	1,12										
12.	Write Runge-Kutta method of fourth order formula.	2	1	2	1,12										
Part-B (3 × 14 = 42 Marks)															
13. a)	Find $f(x)$ by Newton's forward and backward interpolation formulae from the following data:	7	3	1	1,12										
	<table border="1"> <tr> <td>x :</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> </tr> <tr> <td>f(x):</td> <td>-1</td> <td>-1</td> <td>1</td> <td>5</td> </tr> </table>	x :	1	2	3	4	f(x):	-1	-1	1	5				
x :	1	2	3	4											
f(x):	-1	-1	1	5											
b)	If $y_1 = 168, y_7 = 192, y_{15} = 336$, then find y_{10} using Lagrange's interpolation formula.	7	3	1	1,12										

14. a)	Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x=0.0$ from the following data: <table border="1" style="margin: 10px auto; border-collapse: collapse;"> <tbody> <tr> <td style="padding: 2px 5px;">x:</td> <td style="padding: 2px 5px;">0.0</td> <td style="padding: 2px 5px;">0.2</td> <td style="padding: 2px 5px;">0.4</td> <td style="padding: 2px 5px;">0.6</td> <td style="padding: 2px 5px;">0.8</td> <td style="padding: 2px 5px;">1.0</td> </tr> <tr> <td style="padding: 2px 5px;">y:</td> <td style="padding: 2px 5px;">0.0</td> <td style="padding: 2px 5px;">0.12</td> <td style="padding: 2px 5px;">0.48</td> <td style="padding: 2px 5px;">1.10</td> <td style="padding: 2px 5px;">2.0</td> <td style="padding: 2px 5px;">3.20</td> </tr> </tbody> </table>	x:	0.0	0.2	0.4	0.6	0.8	1.0	y:	0.0	0.12	0.48	1.10	2.0	3.20	7	2	2	1,12						
x:	0.0	0.2	0.4	0.6	0.8	1.0																			
y:	0.0	0.12	0.48	1.10	2.0	3.20																			
	b) Apply Taylor's series method to find the approximate value of $y(0.1)$ for the initial value problem $y' = x^2y - 1, y(0) = 1$.	7	3	2	1,12																				
15. a)	A random variable X has the following probability distribution. <table border="1" style="margin: 10px auto; border-collapse: collapse;"> <tbody> <tr> <td style="padding: 2px 5px;">X:</td> <td style="padding: 2px 5px;">0</td> <td style="padding: 2px 5px;">1</td> <td style="padding: 2px 5px;">2</td> <td style="padding: 2px 5px;">3</td> <td style="padding: 2px 5px;">4</td> </tr> <tr> <td style="padding: 2px 5px;">P(X):</td> <td style="padding: 2px 5px;">3k</td> <td style="padding: 2px 5px;">3k</td> <td style="padding: 2px 5px;">K</td> <td style="padding: 2px 5px;">2k</td> <td style="padding: 2px 5px;">6k</td> </tr> </tbody> </table> Find (i) k (ii) $E(X)$ (iii) $Var(X)$ (iv) $P(X < 2)$	X:	0	1	2	3	4	P(X):	3k	3k	K	2k	6k	7	2	3	1,12								
X:	0	1	2	3	4																				
P(X):	3k	3k	K	2k	6k																				
	b) The random variable X is normally distributed with mean 9 and standard deviation 3. Find the probabilities that (i) $X \geq 15$ and (ii) $0 \leq X \leq 9$.	7	3	3	1,12																				
16. a)	A random sample of 7 students had the following I.Q's: 85, 96, 105, 102, 82, 89, 90. Does this data support the claim of a population mean of I.Q 100 ? Test at 5% level of significance.	7	4	4	1,12																				
	b) Two samples of sizes 9 and 8 give the sum of squares of deviations from their respective means equals to 160 and 91 respectively. Can these be regarded as drawn from the same normal population?	7	4	4	1,12																				
17. a)	Use the method of least squares to fit a straight line $y = a + bx$ for the following data: <table border="1" style="margin: 10px auto; border-collapse: collapse;"> <tbody> <tr> <td style="padding: 2px 5px;">x:</td> <td style="padding: 2px 5px;">0</td> <td style="padding: 2px 5px;">2</td> <td style="padding: 2px 5px;">5</td> <td style="padding: 2px 5px;">7</td> </tr> <tr> <td style="padding: 2px 5px;">y:</td> <td style="padding: 2px 5px;">-1</td> <td style="padding: 2px 5px;">5</td> <td style="padding: 2px 5px;">12</td> <td style="padding: 2px 5px;">20</td> </tr> </tbody> </table>	x:	0	2	5	7	y:	-1	5	12	20	7	2	5	1,12										
x:	0	2	5	7																					
y:	-1	5	12	20																					
	b) Compute the coefficient of correlation between X and Y from the following data: <table border="1" style="margin: 10px auto; border-collapse: collapse;"> <tbody> <tr> <td style="padding: 2px 5px;">X</td> <td style="padding: 2px 5px;">1</td> <td style="padding: 2px 5px;">2</td> <td style="padding: 2px 5px;">3</td> <td style="padding: 2px 5px;">4</td> <td style="padding: 2px 5px;">5</td> <td style="padding: 2px 5px;">6</td> <td style="padding: 2px 5px;">7</td> <td style="padding: 2px 5px;">8</td> <td style="padding: 2px 5px;">9</td> </tr> <tr> <td style="padding: 2px 5px;">Y</td> <td style="padding: 2px 5px;">12</td> <td style="padding: 2px 5px;">11</td> <td style="padding: 2px 5px;">13</td> <td style="padding: 2px 5px;">15</td> <td style="padding: 2px 5px;">14</td> <td style="padding: 2px 5px;">17</td> <td style="padding: 2px 5px;">16</td> <td style="padding: 2px 5px;">19</td> <td style="padding: 2px 5px;">10</td> </tr> </tbody> </table> Also, find the regression line of x on y .	X	1	2	3	4	5	6	7	8	9	Y	12	11	13	15	14	17	16	19	10	7	2	5	1,12
X	1	2	3	4	5	6	7	8	9																
Y	12	11	13	15	14	17	16	19	10																
18. a)	Using Newton's divided difference formula, find $f(8)$ and $f(12)$ from the following data: <table border="1" style="margin: 10px auto; border-collapse: collapse;"> <tbody> <tr> <td style="padding: 2px 5px;">x:</td> <td style="padding: 2px 5px;">4</td> <td style="padding: 2px 5px;">5</td> <td style="padding: 2px 5px;">7</td> <td style="padding: 2px 5px;">10</td> <td style="padding: 2px 5px;">11</td> <td style="padding: 2px 5px;">13</td> </tr> <tr> <td style="padding: 2px 5px;">f(x):</td> <td style="padding: 2px 5px;">48</td> <td style="padding: 2px 5px;">100</td> <td style="padding: 2px 5px;">294</td> <td style="padding: 2px 5px;">900</td> <td style="padding: 2px 5px;">1210</td> <td style="padding: 2px 5px;">2028</td> </tr> </tbody> </table>	x:	4	5	7	10	11	13	f(x):	48	100	294	900	1210	2028	7	3	1	1,12						
x:	4	5	7	10	11	13																			
f(x):	48	100	294	900	1210	2028																			
	b) Use Runge-Kutta method of order 4 to find the approximate value of $y(1.2)$ for $y' = x^2 + y^2, y(1) = 2$ with $h = 0.1$.	7	3	2	1,12																				

19.	Answer any two of the following:														
a)	If X is a discrete random variable, then prove that $E(aX+b) = aE(X) + b$ and $Var(aX+b) = a^2 Var(X)$, where a and b are constants.	7	2	3	1,12										
b)	In experiments on pea breeding, the following frequencies of seeds were obtained:	7	3	4	1,12										
	<table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>Round and Yellow</th> <th>Wrinkled and Yellow</th> <th>Round and Green</th> <th>Wrinkled and Green</th> <th>Total</th> </tr> </thead> <tbody> <tr> <td>315</td> <td>101</td> <td>108</td> <td>32</td> <td>556</td> </tr> </tbody> </table>	Round and Yellow	Wrinkled and Yellow	Round and Green	Wrinkled and Green	Total	315	101	108	32	556				
Round and Yellow	Wrinkled and Yellow	Round and Green	Wrinkled and Green	Total											
315	101	108	32	556											
	Theory predicts that the frequencies should be in 9:3:3:1. Examine the correspondence between theory and experiment.														
c)	If $x = 4y + 5$ and $y = kx + 4$ are the regression lines of x on y and y on x respectively, show that $0 < 4k < 1$ and if $k = \frac{1}{20}$, find \bar{x} and \bar{y} .	7	3	5	1,12										

M: Marks; L: Bloom's Taxonomy Level; CO: Course Outcome; PO: Programme Outcome

S. No.	Criteria for questions	Percentage
1	Fundamental knowledge (Level-1 & 2)	44.88
2	Knowledge on application and analysis (Level-3 & 4)	55.12
3	*Critical thinking and ability to design (Level-5 & 6) (*wherever applicable, subject to a maximum of 10%)	0
